USN [redacted]

18MAT21

Second Semester B.E. Degree Examination, Dec.2019/Jan.2020
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) along $2\bar{i} - 3\bar{j} + 6\bar{k}$. **(06 Marks)**

b. If $\bar{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find div $\bar{f}$ and curl $\bar{f}$. **(07 Marks)**

c. Find the constants a and b such that $\bar{F} = (axy + z^3)\bar{i} + (3x^3 - z)\bar{j} + (bxz^2 - y)\bar{k}$ is irrotational. Also find a scalar potential ϕ if $\bar{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. If $\bar{F} = xy\bar{i} + yz\bar{j} + zx\bar{k}$ evaluate $\int_C \bar{F}.d\bar{r}$ where C is the curve represented by x = t, y = t^2, $z = t^3$, $-1 \le t \le 1$. **(06 Marks)**

b. Using Stoke's theorem Evaluate $\oint_C \bar{F}.d\bar{r}$ if $\bar{F} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ taken round the rectangle bounded by x = 0, x = a, y = 0, y = b. **(07 Marks)**

c. Using divergence theorem, evaluate $\iint_S \bar{F}.\hat{n}\,ds$ if $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken around $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. **(07 Marks)**

Module-2

3 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$ **(06 Marks)**

b. Solve $(D^2 + 4D + 3)y = e^{-x}$ **(07 Marks)**

c. Using the method of variation of parameter solve y″ + 4y = tan2x. **(07 Marks)**

OR

4 a. Solve $(D^3 - 1)y = 3\cos 2x$ **(06 Marks)**

b. Solve $x^2y'' - 5xy' + 8y = 2\log x$ **(07 Marks)**

c. The differential equation of a simple pendulum is $\frac{d^2x}{dt^2} + W_0^2 x = F_0 \sin nt$, where W_0 and F_0 are constants. Also initially x = 0, $\frac{dx}{dt} = 0$ solve it. **(07 Marks)**

Module-3

5 a. Find the PDE by eliminating the function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given $\frac{\partial z}{\partial y} = -2\sin y$, when x = 0 and z = 0, when y is odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Derive one-dimensional wave equation in usual notations. **(07 Marks)**

OR

6 a. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$ $\frac{\partial z}{\partial x} = a \sin y$ and $z = 0$. **(06 Marks)**

b. Solve $x(y - z)\, p + y\,(z - x)\, q = z\,(x - y)$. **(07 Marks)**

c. Find all possible solution of $U_t = C^2 U_{xx}$ one dimensional heat equation by variable separable method. **(07 Marks)**

Module-4

7 a. Test for convergence for

$$1 + \frac{2!}{2^2} + \frac{3!}{3^2} + \frac{4!}{4^2} + \ldots\ldots\ldots$$ **(06 Marks)**

b. Find the series solution of Legendre differential equation $(1 - x^2)y'' - 2xy' + n(n + 1) = 0$ leading to $P_n(x)$. **(07 Marks)**

c. Prove the orthogonality property of Bessel's function as

$$\int_0^1 x\, \bar{j}_n(\alpha x)\, \bar{j}_n(\beta x) dx = 0 \quad \alpha \neq \beta$$ **(07 Marks)**

OR

8 a. Test for convergence for

$$\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$ **(06 Marks)**

b. Find the series solution of Bessel differential equation $x^2y'' + xy' + (n^2 - x^2)\, y = 0$ Leading to $\bar{j}_n(x)$. **(07 Marks)**

c. Express the polynomial $x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. **(07 Marks)**

Module-5

9 a. Using Newton's forward difference formula find f(38). **(06 Marks)**

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304

b. Find the real root of the equation $x \log_{10} x = 1.2$ by Regula falsi method between 2 and 3 (Three iterations). **(07 Marks)**

c. Evaluate $\int_4^{5.2} \log x\, dx$ by Weddle's rule considering six intervals. **(07 Marks)**

OR

10 a. Find f(9) from the data by Newton's divided difference formula:

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

(06 Marks)

b. Using Newton – Raphson method, find the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$. **(07 Marks)**

c. By using Simpson's $\left(\frac{1}{3}\right)$ rule, evaluate $\int_0^6 \frac{dx}{1 + x^2}$ by considering seven ordinates. **(07 Marks)**

* * * * *